

NAG Toolbox for MATLAB

s11ab

1 Purpose

s11ab returns the value of the inverse hyperbolic sine, $\operatorname{arcsinh} x$, via the function name.

2 Syntax

```
[result, ifail] = s11ab(x)
```

3 Description

s11ab calculates an approximate value for the inverse hyperbolic sine of its argument, $\operatorname{arcsinh} x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\operatorname{arcsinh} x = x \times y(t) = x \sum_{r=0}^{\infty} c_r T_r(t), \quad \text{where } t = 2x^2 - 1.$$

For $|x| > 1$ it uses the fact that

$$\operatorname{arcsinh} x = \operatorname{sign} x \times \ln\left(|x| + \sqrt{x^2 + 1}\right).$$

This form is used directly for $1 < |x| < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $|x| \geq 10^k$, $\sqrt{x^2 + 1}$ is equal to $|x|$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arcsinh} x = \operatorname{sign} x \times (\ln 2 + \ln|x|).$$

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{1+x^2} \operatorname{arcsinh} x} \delta \right|.$$

That is, the relative error in the argument, x , is amplified by a factor at least $\frac{x}{\sqrt{1+x^2} \operatorname{arcsinh} x}$, in the result.

The equality should hold if δ is greater than the *machine precision* (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

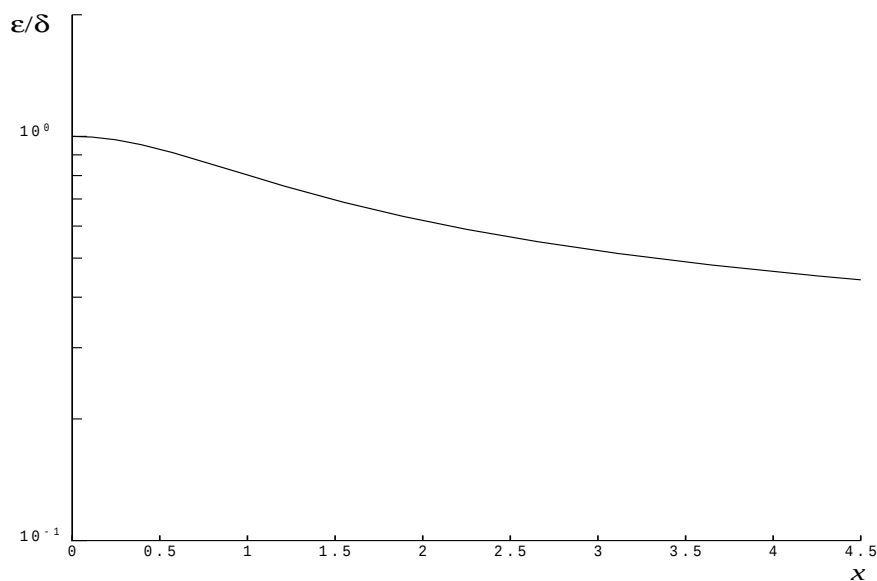


Figure 1

It should be noted that this factor is always less than or equal to one. For large x we have the absolute error in the result, E , in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by *machine precision*.

8 Further Comments

None.

9 Example

```
x = -2;  
[result, ifail] = s11ab(x)  
  
result =  
    -1.4436  
ifail =  
         0
```
